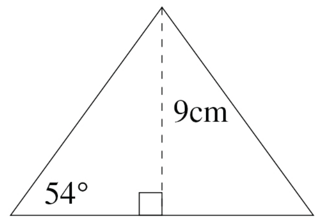
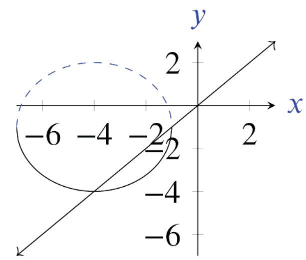
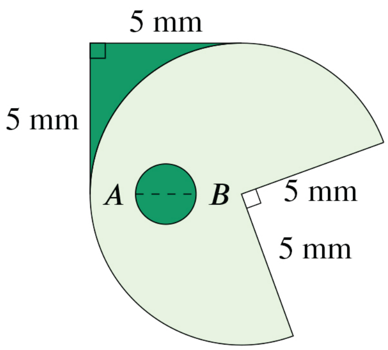
Math & Science

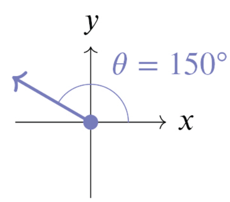
Introductory - Advanced Aptitude Development

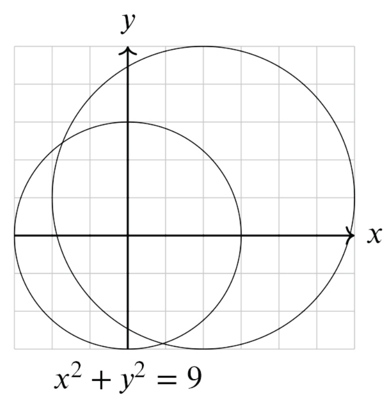
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

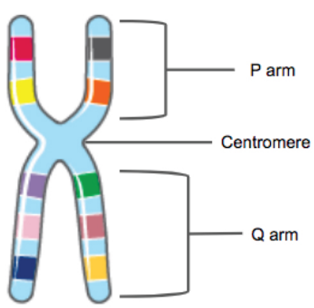


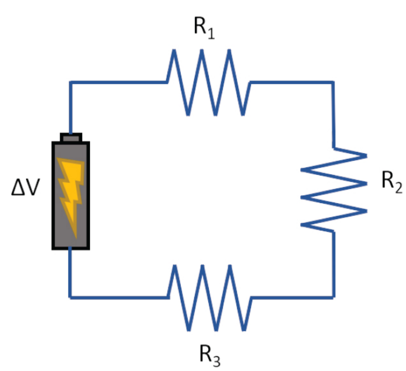


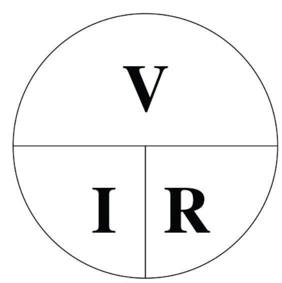


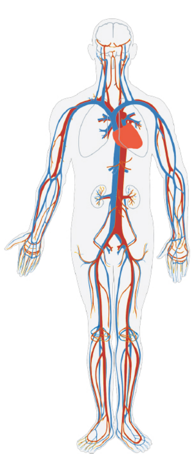


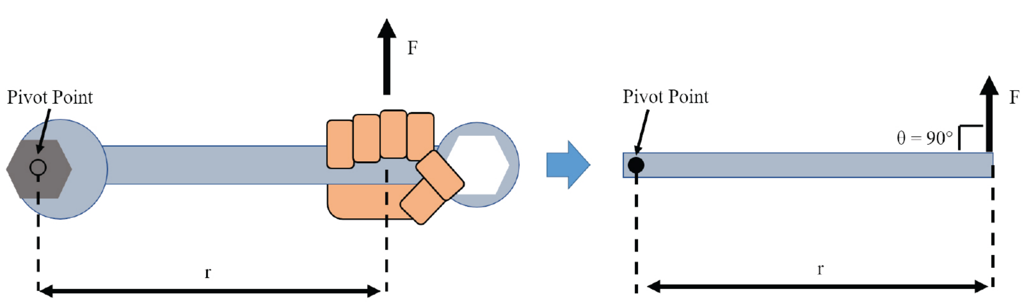












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Math & Science

Introductory - Advanced Aptitude Development

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Preface

There are many talented athletes in the sports world; the one thing they all have in common is strength and conditioning training. It does not matter how talented an athlete is; their performance is improved with that training. Similarly, math and science are like strength and conditioning training for industry workers; it does not matter how talented we are; we are improved by employing that training.

Fortune 500 companies prefer CEOs with engineering degrees because engineers specialize in problem solving. The core curriculum for engineering majors is math and science based. A healthy aptitude in math and science, among other things, conditions the mind to formulate actions that will yield desired outcomes when challenges arise. The more math and science we are exposed to, the better we can make good decisions in response to challenges. Academic problem solving practice translates into real world problem solving capabilities.

The hope is that this book will be another tool on our tool belt that we can use to get the job done right. Just like how athletes must undergo continual training to perform at the highest levels, we must also continue to train our minds to excel in our performance. Let the journey begin!

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Contents

Proper and Improper Fraction Division

Dividing by a proper or improper fraction is similar to the multiplication process. However, there is an additional step that must be performed when solving division problems. Dividing by a number is the same as multiplying by that number's reciprocal. In the case of proper and improper fractions, generating a reciprocal is achieved by switching the numerator and denominator. Then multiply as is done with the multiplication of proper and improper fractions.

Examples

1.) $\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \cdot \frac{9}{4} = \frac{3}{2} = 1\frac{1}{2}$

2.) $\frac{2}{\frac{4}{7}} = 2 \div \frac{4}{7} = 2 \cdot \frac{7}{4} = \frac{7}{2} = 3\frac{1}{2}$

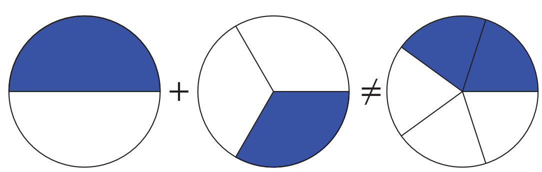
1.2.4 Practice

1) $\frac{5}{6} \div \frac{3}{4} =$ **2)** $\frac{\frac{2}{9}}{\frac{8}{15}} =$ **3)** $\frac{12}{49} \div \frac{21}{25} =$ **4)** $\frac{3}{7} \div \frac{28}{9} =$ **5)** $\frac{4}{7} \div \frac{9}{11} =$ **6)** $\frac{2}{36} \div \frac{23}{45} =$

7) $\frac{8}{9} \div \frac{33}{14} =$ **8)** $\frac{7}{8} \div \frac{9}{16} =$ **9)** $\frac{\frac{6}{27}}{\frac{16}{30}} =$ **10)** $\frac{12}{25} \div \frac{14}{15} =$

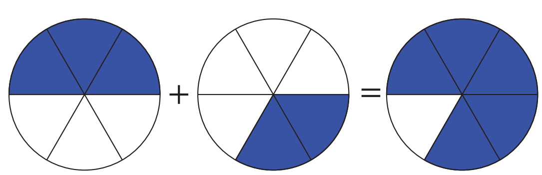
Fraction Addition

Fractions can combine to form new fractions or whole integers. However, fraction addition does not work the same way as integer addition. To illustrate this point examine the figure below.



$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{5}$

To add fractions, the number of divisible parts (the denominators) have to match for each fraction. So in the previous example, each half could be cut into thirds to make six parts, and each third could be cut into halves to make six parts in the second fraction.



$\frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}$ $\frac{1}{3} = \frac{1\times2}{3\times2} = \frac{2}{6}$ $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Ratio and Proportion Fundamentals

Ratio and Proportion Basics

It is often useful to compare two different values. Like determining which city has a better crime rate and by how much? How many teachers are there for every student in the class? What is the comparison between the company's debt to its equity? Which job has better pay per hour?

A ratio is a way to compare two numbers. Ratios are sometimes written as fractions, and other times they are separated with a ":" or "to." If there is one teacher for every eight students, the teacher to student ratio is either $\frac{1}{8}$, 1:8, or 1 to 8. Likewise, if there are 20 scholarships available and 35 students qualified, the scholarship to students ratio is $\frac{20}{35}$, 20:35 or 20 to 35. Ratios should be reduced to their simplest form, in this case, $\frac{4}{7}$, 4:7, or 4 to 7.

Example

Super flyweight boxer Juan Fernandez Pablo has fought 28 times in his career; he won 21 times with no draws. What is his career win to loss ratio?

$$\frac{\text{wins}}{\text{losses}} \quad \frac{21}{7} \div \frac{7}{7} = \frac{3}{1}$$ His career win/loss ratio is 3:1.

Two equivalent ratios are called proportional. Proportions are typically expressed as two equivalent fractions separated by an equal sign. Geometry provides visualization for ratios and proportions.

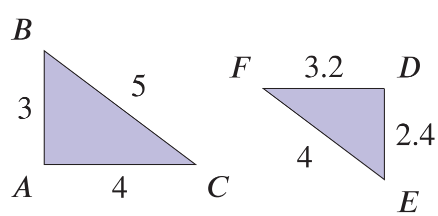
Examples

1) The ratio of *a* to *b* is 5 to 7.

$$\frac{a}{b} = \frac{5}{7}$$

2) The proportions of the lengths of corresponding sides in similar triangles are all equal.

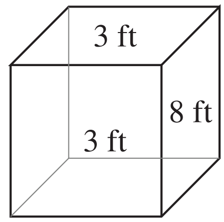
$$\triangle ABC \sim \triangle DEF \rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$



$$\text{e.g. } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 1.25$$

Volume

Volume adds yet another dimension that can be analyzed. In the closet example, there was a length of 3 feet and a width of 3 feet. When the additional dimension of height is added, we can determine how much space is available to be occupied. If the height of the closet is 8 feet, using the formula ($V = lwh$) we can calculate that the volume of the closet is 72 ft^3.



Again, multi-dimensional analysis can be utilized when there is a need to convert from one unit to another desired unit. Continuing with the closet example, the volume of the closet is 72 ft^3; there is a need to know how many cubic yards of material can be stored in the closet. This is accomplished by following the same process as multi-dimensional analysis for area, except the equivalent numerator and denominator are cubed. When units are cubed, converting those units requires multiplying by a cube. The multi-dimensional analysis of 72ft^3 in terms of yd^3 works out as follows:

$$72\text{ft}^3\left(\frac{1\text{ yd}}{3\text{ft}}\right)^3 = 72\text{ft}^3\left(\frac{1\text{ yd}^3}{27\text{ft}^3}\right) = 2.7\text{yd}^3$$

Example

John is struggling with redoing his math homework. He was asked to convert 1 cubic inch into cubic feet. He made the mistake of thinking one foot is equivalent to 6 inches when a foot is 12 inches. What was John's original answer, and what should have been his answer?

What John thought was:

$$\left(\frac{\text{foot}}{6\text{ inches}}\right)^3 = \left(\frac{\text{foot}^3}{216\text{ inches}^3}\right)$$

so he gave the answer of 216 cubic inches per cubic foot.

The real answer is based on the fact that one foot is equivalent to 12 inches, which by a similar process yields one cubic foot is equivalent to 1,728 cubic inches.

3.4.2 Practice

1) Write 1 m^3 per freezer in terms of L per freezer. (1L = 1,000 cm^3)

2) Write 4 containers per ft^3 in terms of containers per yd^3.

3) Write one bacterium every 2 μm^3 in terms of bacteria per cm^3.

4) 45 in^3 in terms of cm^3 is?

English to Algebra Translation	
a number increased by 7	$x + 7$
a number reduced by 6	$x - 6$
3 times a number	$3x$
a number divided by 5	$\frac{x}{5}$
one third of a number	$\frac{1}{3}x$
4 times a number increased by 3	$4x + 3$
one number minus another number	$x - y$
the square of a number times two	$2x^2$

Example

1) A car manufacturer is going to produce x number of cars. They need to order tires for the cars that will be produced. According to inventory, 17004 tires are already in their warehouse. Write an algebraic expression to determine the number of tires that need to be ordered.

4x-17,004

4.3 Practice

Translate the following to an algebraic expression:

1) decreasing a number by 5

2) the product of six times a number increased by 8

3) 15 more than a number plus 3 times another number

4) the quotient of 12 and a number minus 3

5) the difference of -6 multiplied by a number and another number squared increased by 11

Translate the following to English:

6) n+18

7) 6x

8) $\frac{x}{5} - 8$

9) 13x-7 + $y^2 - 4$

10) $\frac{1}{8}x - 4$

Point-Slope Form

There are many situations in which it is useful to describe the mathematical equation for a line. A line's equation describes how steep the line is, where it starts, and how two variables interact.

A linear function has a very straight graph, such that the two variables change together at a consistent rate across the whole graph. Linear functions are the most straightforward functions to draw, describe, graph, and understand.

The point-slope form of a graph is the most useful for solving for slope or finding a certain point. It is beneficial for any problem that involves a graph and solving for a value.

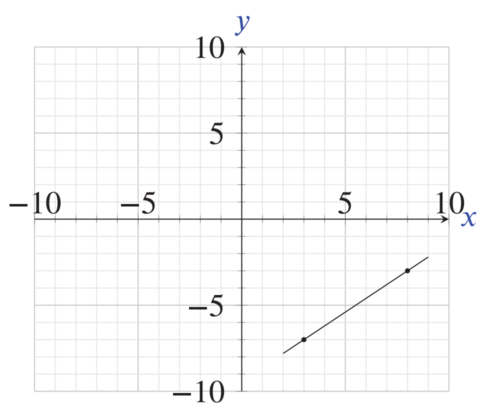
Because the slope of a line never changes, knowing the slope of a line and one point on the line is enough to write an equation for that line using the slope formula. Remove the subscripts for the second point, so they become just y and x.

$$\frac{y - y_1}{x - x_1} = m \rightarrow y - y_1 = m(x - x_1)$$

This formula is handy, which is why it is a rare visitor to multiple choice exams and answer keys: it presents an infinite number of solutions for any given line. Often one will be implicitly or directly asked to solve for y and simplify. However, point-slope form is often the fastest way to generate the equation when only one point is known or when the line has a restricted domain (because the real world is full of line segments that do not necessarily cross any axis lines) for a line. Sometimes the graphing point-slope form is faster relative to other forms of lines because it can target the most convenient point for graphing while the other forms are too rigid. Note: Be careful with signs. The formula has two sign changes.

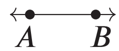
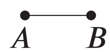
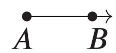
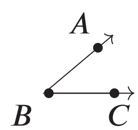
Examples

1) Find an equation for the line segment in the graph.



First, find convenient points to work with. The two points identified from the graph are $(3, -7)$ and $(8, -3)$.

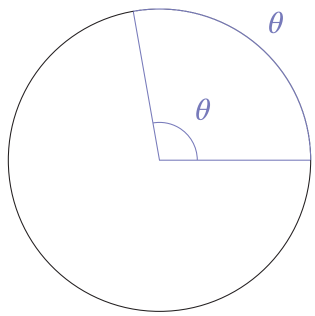
Line and angle terminology

Term	Definition	Diagram	Notation or Sentence
Point	A zero dimensional object used for location.	• A	A
Line	A one dimensional object of infinite length.		$\overleftrightarrow{AB}$
Line segment	A finite portion of a line.		$\overline{AB}$
Ray	Half of a line.		$\overrightarrow{AB}$
Angle	Two rays coming out from the same point.		$\angle B$ or $\angle ABC$ or $\angle CBA$
Degree	The angle or arc measure corresponding to $\frac{1}{360}$ of a circle.		$1°$
Minute	The angle or arc measure corresponding to $\frac{1}{60}$ of a degree.	Invisible at this scale.	$0°1'$
Second	The angle or arc measure corresponding to $\frac{1}{60}$ of a minute.	Invisible at this scale.	$0°0'1''$
Radian	The angle or arc measure corresponding to one radius length along a circle.		1 rad. Radian measures often contain π and then may go with other units.

Radians and DMS

Humans measure items all the time. Height is measured year after year as a child grows. Weight is used to measure flour, water, salt, and yeast in a loaf of bread. Speed is measured in a car to determine how fast we are going. Angles are measured to determine a direction or location. As with all measurements, there is a standard way to measure angles.

Either radians or degrees can be used to determine the size of an angle or the size of an arc length, which is a portion of a circle. When these units are used to measure arcs, the arc measurement equals the matching central angle measure.



Converting between radians and degrees requires knowing that there are 2π radians in a circle, and 360° in a circle. The conversion will be using either $\frac{\pi}{180°}$ or $\frac{180°}{\pi}$, memorizing the reduced fractions is not as important as remembering the underlying concept.

Examples

1) Convert $\sqrt{2}$ radians to an angle rounded to the nearest second using a calculator.

$$\sqrt{2}\left(\frac{180°}{\pi}\right) \approx 81.02846845°$$

Multiply the decimal portion by 60 to get the minutes.

$$0.02846845° \approx 1.708107248'$$

Multiply the decimal portion of this to get the seconds.

$$0.708107248' = 42.4864349''$$

Rounding as directed, the final answer is $81°1'42''$.

2) Convert $100°45'20''$ into radians.

$$\left(100 + \frac{45}{60} + \frac{20}{3600}\right)\left(\frac{\pi}{180°}\right)^{\circ} = \frac{4534\pi}{8100} = \frac{2267\pi}{4050}$$

9.1 Practice

(Answers to practice begins on page 400)

1) Convert $40°40'40''$ into radians.

2) Using a calculator, convert 1 radian to an angle rounded to the nearest second.

Spread

The term spread refers to how closely packed or loosely packed a data set happens to be. The quartiles are the specific benchmarks within a sorted data set that give a quick way of summarizing the shape of a data set.

- Q_0 is the minimum point in the data set.
- Q_1 is the median of the data subset that goes from the minimum to the median of the original set and is called the first quartile.
- Q_2 is the median of the original set.
- Q_3 is the median of the data subset that goes from the median to the maximum of the original data set and is called the third quartile.
- Q_4 is the maximum of the original data set.

To visualize this, imagine having a number line ranging from 0 to 8 so $Q_0 = 0$, $Q_1 = 2$, $Q_2 = 4$, $Q_3 = 6$, $Q_4 = 8$. The idea is that between any Q_n and Q_{n+1} is 25% of the original data set.

The interquartile range, known as IQR for short in many places, is simply $Q_3 - Q_1$. It is a way to measure the spread of the middle 50% of the data.

Standard deviation is an incredibly useful number in statistics. The standard deviation describes how the data points in a data set vary from the mean value, or how far each data point *deviates* from the *standard* (or mean).

If all the data points are close to the mean, the standard deviation is small. If all the data points are far from the mean, the standard deviation is large. Standard deviation is a critical concept because a study with a large standard deviation is not considered scientifically valid. After all, the numbers vary too much to show a clear pattern or draw a strong conclusion. While important, standard deviation has a tedious calculation:

$$s_x = \sqrt{\frac{\sum_{i=1}^{i=n}(x_i - \overline{x})^2}{n-1}}$$

Here, s_x stands for standard deviation, x_i stands for an individual data point, $\overline{x}$ stands for the mean of the data, and n stands for the number of data points in the sample or population.

To find the standard deviation, find the square of the difference between each data point and the mean, then add all of those values together. Divide by the number of values in the sample minus 1 and finally take the square root. For a large data set, this takes an extremely long time.

For example, consider a sample:

$$x \in \{2, 3, 3, 4\}$$

Science Fundamentals

Scientific Terminology

Professionals in the world of science use terms that can seem intimidating to those who are not familiar with scientific terminology. Having a solid knowledge of key scientific terms can aid in the understanding of scientific information. Below is a list of common scientific terms that you may encounter:

Figure 6: Scientific Terms

Term	Definition
Cause	Events that provide the generative force of something.
Conclusion	A position or opinion reached after considering and analyzing evidence.
Context	The set of facts or circumstances that surround a problem or situation.
Control group	Group that is randomly chosen not to receive the experimental treatment.
Control variable	A variable that does not change during an experiment.
Dependent variable	Variable that responds to the independent variable. Dependent variable is what is being measured in the experiment.
Double-blind	Neither the researcher nor the subject knows if the subject is receiving the treatment or a placebo.
Effect	A result that is caused by a previous phenomenon.
Evaluate	Estimate the nature, quality, ability or significance of a problem, idea, or situation.
Experimental group	Subjects randomly chosen to receive the experimental treatment.
Fact	Information about events that have occurred that is supported by evidence.
Hypothesis	A prediction that serves as the starting point for an investigation.
Independence	One factor does not have influence on another.
Infer	Conclude by reasoning.
Inference	Drawing a conclusion on the basis of circumstantial evidence.
Opinion	A belief or judgement.
Placebo	Artificial treatment that should have no real effect.
Scientific Law	Descriptions of natural phenomena that describe an observation.
Scientific Theory	An explanation of an idea based on observations, explanations, and reasoning.

Source: "Scientific Method Vocabulary Terms."

Ecosystem

There are many processes that organisms undergo in order to sustain life. All of these processes occur at the cellular level in either plants, animals, or other microorganisms. Plants are producers that use sunlight and water to convert carbon dioxide to carbohydrates (sugar) through a process known as photosynthesis. Photosynthesis is a vital process to maintain balance in an ecosystem because plants are able to take waste gases like carbon dioxide and facilitate the production of oxygen, which is utilized by almost all living organisms.

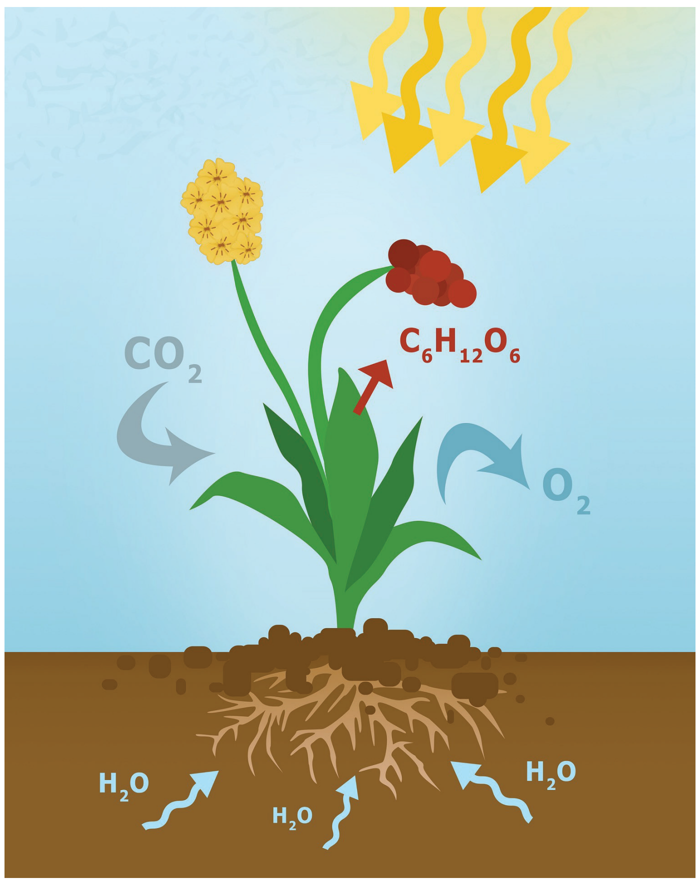


Figure 30. Photosynthesis process

Another cellular process that is important to the sustainability of life is fermentation. Fermentation is an anaerobic (without oxygen) metabolic process in which microorganisms like bacteria and yeast convert sugar to gases, alcohol, or acids. Muscle cells of the human body can also go through the fermentation process producing lactic acid. This build up of lactic acid occurs when muscles are exposed to high intensity exercises and are deprived of oxygen. A person can feel this through muscle soreness or cramps.

(d) 2000 rev/min^2

3. A satellite orbiting the planet does due to centripetal acceleration. What force causes the centripetal force to keep The satellite in planetary orbit?

   (a) Kinetic Friction

   (b) Normal

   (c) Gravitational

   (d) Elastic

Torque

Torque τ is the amount of force required to cause a rotation and is commonly referred to the turning force. Consider a wrench unscrewing a bolt as depicted in figure 44. The wrench acts as a lever arm or rigid rod where the pivot point is centered around the bolt. The force is applied perpendicular to the lever arm. If the force is applied parallel to the lever area, i.e., pushing into the wrench length-wise, the bold will not turn. Where you place your hand on the wrench will affect the amount of torque you apply to unscrew the bolt.

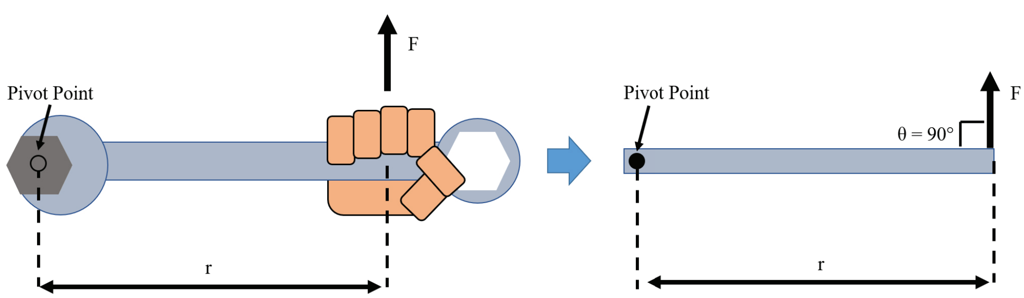


Figure 44. wrench

Formally torque is written with the Greek letter τ and mathematically written as the product of the distance from the pivot point (r), the force applied (F) to create the torque, and the (sin) of the angle between the lever arm and the direction of the force, θ.

$$\tau = rF\sin\theta \tag{16}$$

From equation 16 we can deduce that the units for torque is a meter Newton, mN. Note: $\sin\theta$ is unit less.

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